

BELLCOMM, INC.

SUBJECT: Sensitivities for Transfer
from Circular to Highly Elliptic
Parking Orbits - Case 103-2

DATE: August 26, 1966

FROM: H. B. Bosch

ABSTRACT

One proposed way of starting on a planetary flight is to assemble and store spacecraft stages in a highly elliptic geocentric orbit. Escape from such an orbit could then be accomplished with relatively small amounts of fuel. However, rendezvous between a vehicle in such an orbit and a shuttle in a circular orbit would be very sensitive to errors in perigee velocity. This memorandum presents a quantitative analysis of the sensitivities of such maneuvers.

(NASA-CR-152968) SENSITIVITIES FOR TRANSFER
FOR CIRCULAR TO HIGHLY ELLIPTIC PARKING
ORBITS (Bellcomm, Inc.) 8 p

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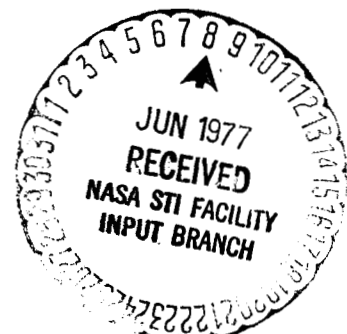
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MEMORANDUM FOR FILE

INTRODUCTION

A shuttle vehicle in a circular Earth orbit of altitude h_p is to rendezvous with a vehicle in an elliptic orbit whose perigee is tangent to the circle (Figure 1). The transfer maneuver, which must occur at perigee, consists of an increase in velocity only. Because a highly eccentric orbit has a perigee velocity, V_p , close to escape velocity, $V_e = V_c \sqrt{2}$, an error in the transfer velocity could cause large errors in orbital period and apogee radius, making rendezvous extremely costly. Should escape conditions be reached rendezvous would, of course, become impossible.

Analysis

The apogee radius of the elliptic parking orbit is related to the perigee radius and velocity as follows

$$r_a = r_p \frac{v_p^2}{v_e^2 - v_p^2} \quad (1)$$

and the period is given by

$$T^2 = \frac{1}{\mu} \left(\frac{r_a + r_p}{2} \right)^3 = \frac{T_c^2}{8} \left(\frac{v_e^2}{v_e^2 - v_p^2} \right)^3 \quad (2)$$

where the period of the circular orbit of radius r_p is given by

$$T_c^2 = \frac{r_p^3}{\mu} \quad (3)$$

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The relative error in apogee radius can now be expressed in terms of the relative error in perigee velocity as

$$\frac{\delta r_a}{r_a} = \frac{V_p}{r_a} \frac{dr_a}{dV_p} \frac{\delta V_p}{V_p} = 2 \frac{V_e^2}{V_e^2 - V_p^2} \frac{\delta V_p}{V_p} \quad (4)$$

Similarly, the relative error in period is

$$\frac{\delta T}{T} = \frac{V_p}{T} \frac{dT}{dV_p} \frac{\delta V_p}{V_p} = 3 \frac{V_p^2}{V_e^2 - V_p^2} \frac{\delta V_p}{V_p} \quad (5)$$

With the aid of (2) these can be written as

$$\frac{\delta r_a}{r_a} = 4 \left(\frac{T}{T_c} \right)^{2/3} \frac{\delta V_p}{V_p} \quad (6)$$

$$\frac{\delta T}{T} = 3 \left[2 \left(\frac{T}{T_c} \right)^{2/3} - 1 \right] \frac{\delta V_p}{V_p} \quad (7)$$

NUMERICAL RESULTS AND EXAMPLE

For a 100 nautical mile altitude circular orbit,

$$r_p = 3,541.66 \text{ NM}$$

$$T_c = 0.06125 \text{ da}$$

$$V_c = 25,567 \text{ ft/sec}$$

$$V_e = 36,157 \text{ ft/sec}.$$

The relative error ratios as given by (6) and (7) are plotted on Figure 2 as functions of T . From these one can obtain the sensitivities of maneuvers for transferring from a circular orbit (100 NM altitude) to an elliptic orbit of any given period.

As an example consider a maneuver for transferring from a 100 NM altitude circular orbit to an elliptic orbit with a period of two days. Assuming a maximum relative velocity error of 0.1%, from Figure 2 we get

$$\frac{\delta T}{T} = 58 \frac{\delta V_p}{V_p} = 0.058 ,$$

$$\frac{\delta r_a}{r_a} = 42 \frac{\delta V_p}{V_p} = 0.042 .$$

Figure 3 shows that an orbit with a two day period has an apogee radius $r_a = 68,000$ NM. Thus, for this maneuver

$$\delta T = 2.8 \text{ hrs.}$$

$$\delta r_a = 2,860 \text{ NM} .$$

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1021-HBB-jdc

Attachment
List of Symbols

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LIST OF SYMBOLS

Symbol

r	Radial distance from center of Earth
T	Orbital period
V	Velocity
δ	Error
μ	Gravitational parameter for Earth

Subscript

a	Apogee
c	Circular
e	Escape
p	Perigee

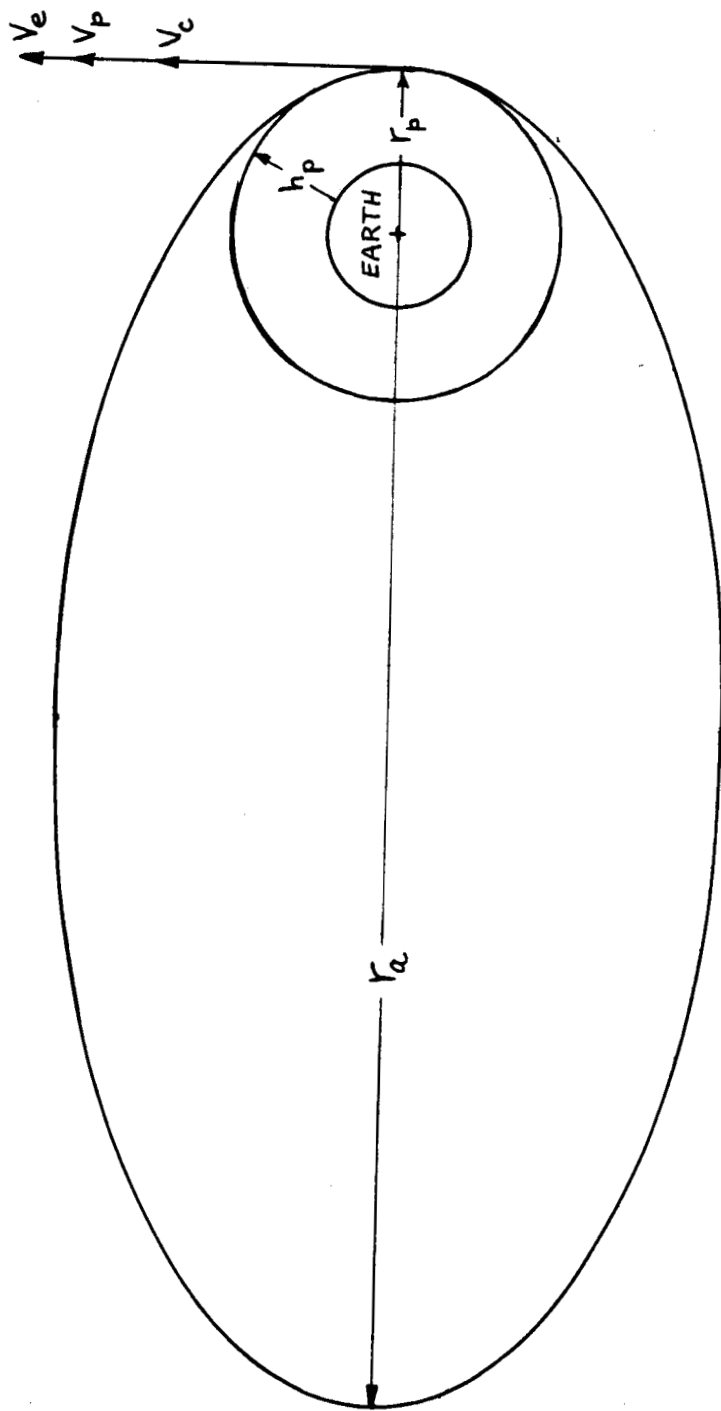


FIGURE 1

TANGENT ELLIPTIC AND CIRCULAR GEOCENTRIC ORBITS

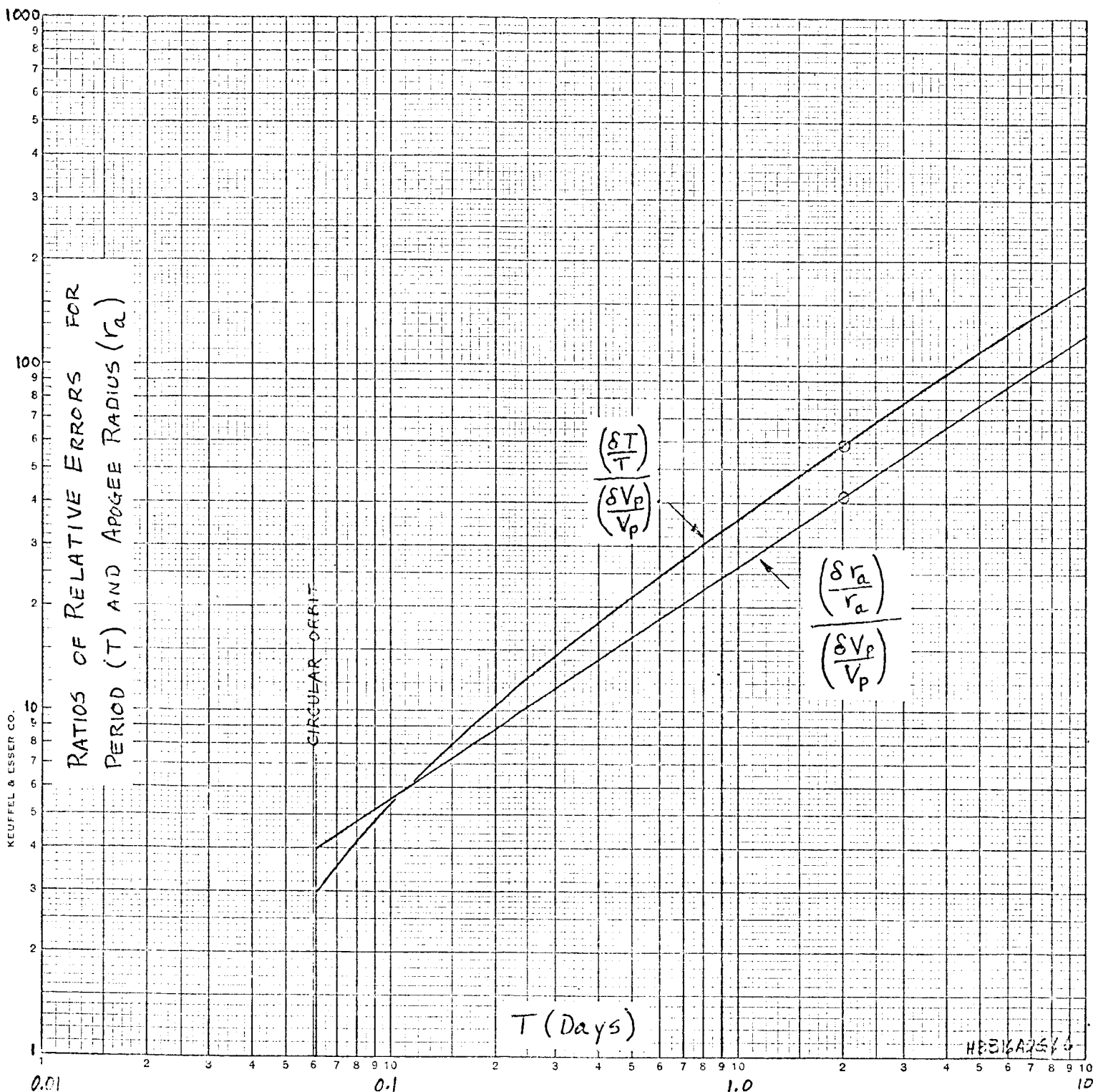


FIGURE 2

RELATIVE ERROR RATIOS IN PERIOD (T) AND APOGEE RADIUS (r_a)

FOR ELLIPTIC ORBITS OF 100 NM PERIGEE ALTITUDE

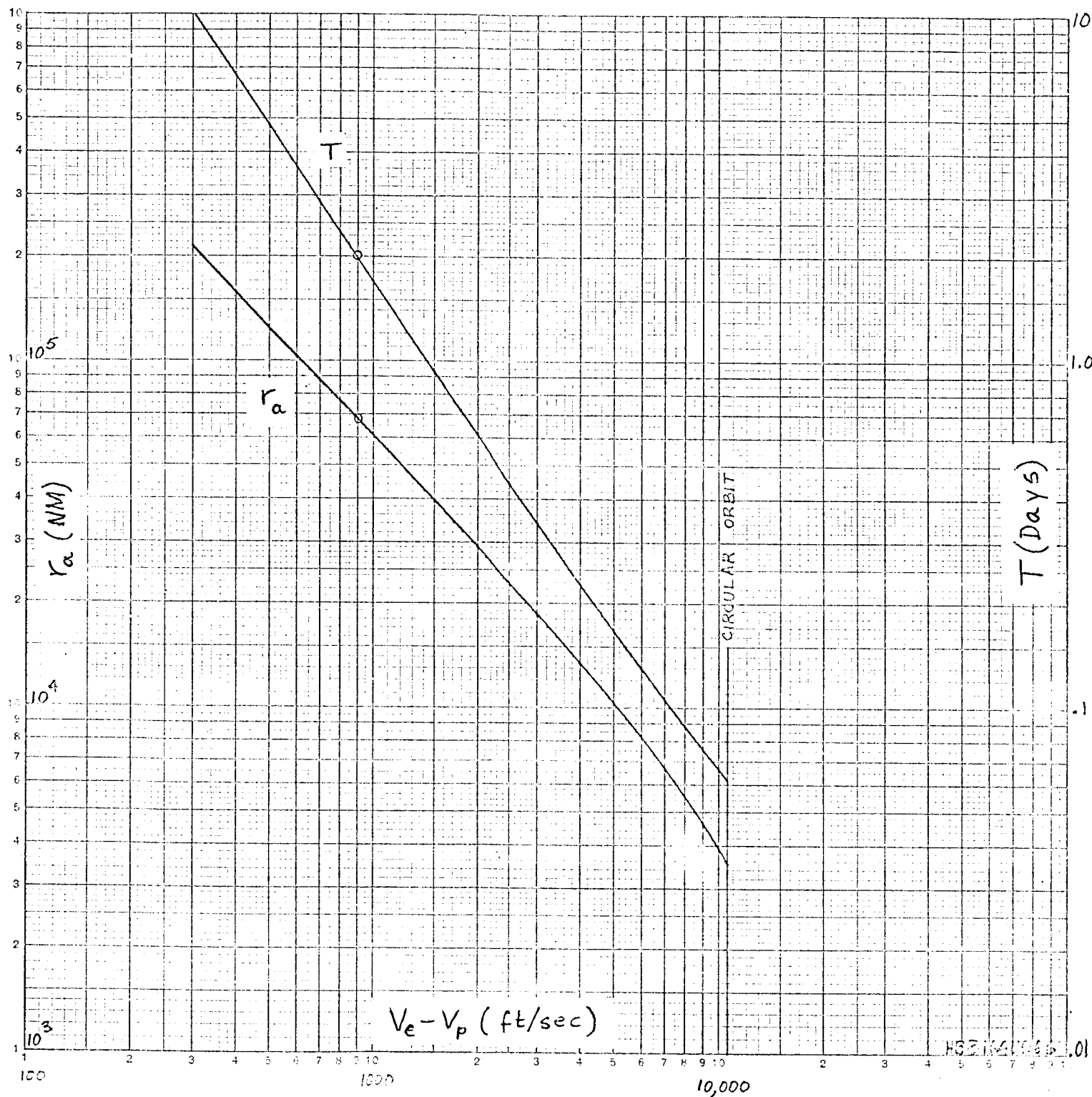


FIGURE 3
PERIOD (T) AND APOGEE RADIUS (r_a) FOR ELLIPTIC
ORBITS OF 100 NM PERIGEE ALTITUDE